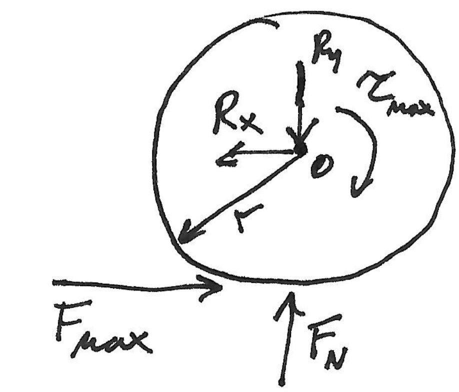
a) Max traction force possible (assuming no slip) ①

FBD wheel



(assuming no rolling resistance)

$$\circlearrowleft + \; \sum \tau_o = 0 = F_{max} r - \tau_{max} = 0$$

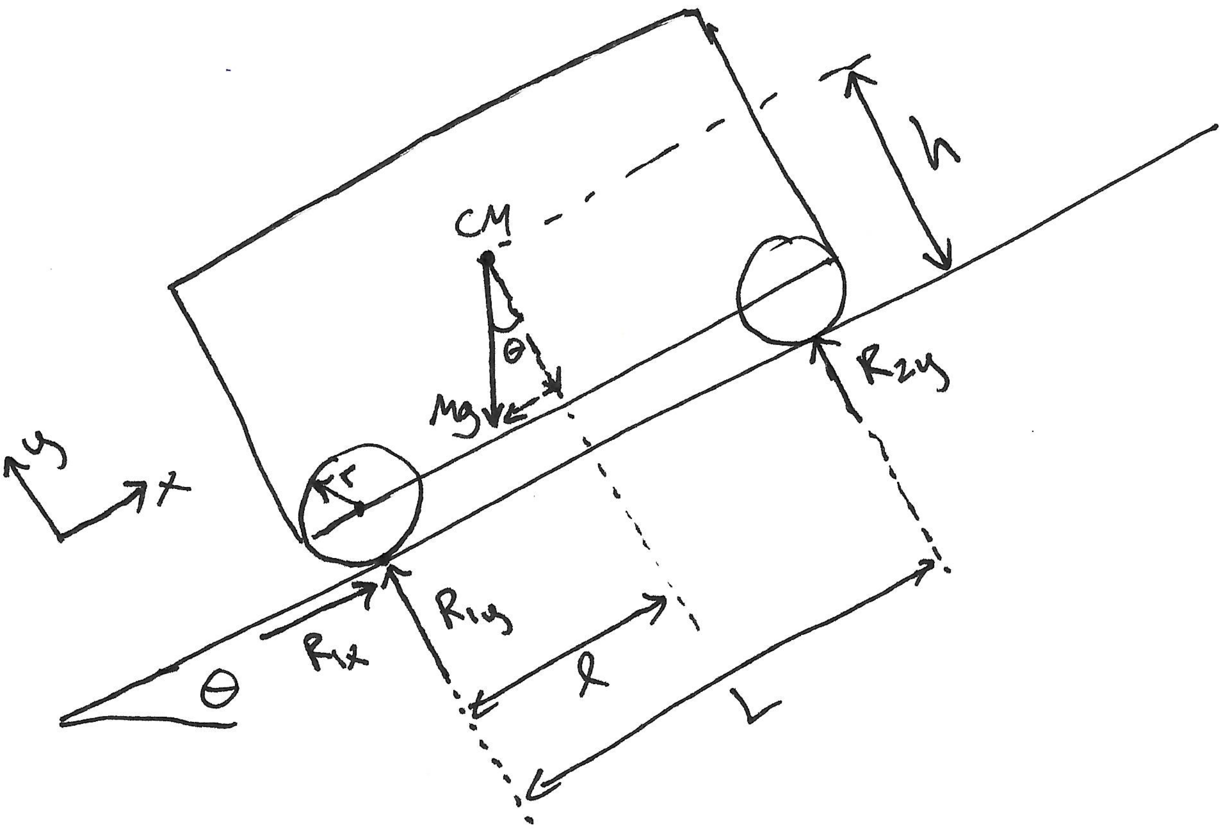
$$F_{max} = \frac{\tau_{max}}{r}$$

For 2-wheel drive

$$F_{max_{total}} = \frac{2\tau_{max}}{r}$$

where τ_{max} is max motor torque

b) Before acceleration:



For wheels to not slip initially:

$$\mu R_{1y} > \frac{2\tau_{max}}{r} \Rightarrow$$ means friction force can withstand the force due to wheel torque

Find dimensions of vehicle to make no-slip inequality true. ⇒ find R_{1y} in terms of h & l

$$\Sigma F_y = 0 = R_{1y} + R_{2y} - Mg\cos\theta \Rightarrow R_{1y} = Mg\cos\theta - R_{2y}$$

$$\Sigma M_{①} = 0 = R_{2y} L - Mg\cos\theta\, l + Mg\sin\theta\, h \Rightarrow R_{2y} = \frac{Mg(\cos\theta\, l - \sin\theta\, h)}{L}$$

combine: $R_{1y} = Mg\cos\theta - \dfrac{Mg(\cos\theta\, l - \sin\theta\, h)}{L}$

plug into no-slip condition:

$$\mu Mg\left[\cos\theta - \frac{(\cos\theta\, l - \sin\theta\, h)}{L}\right] > \frac{2\tau_{max}}{r}$$

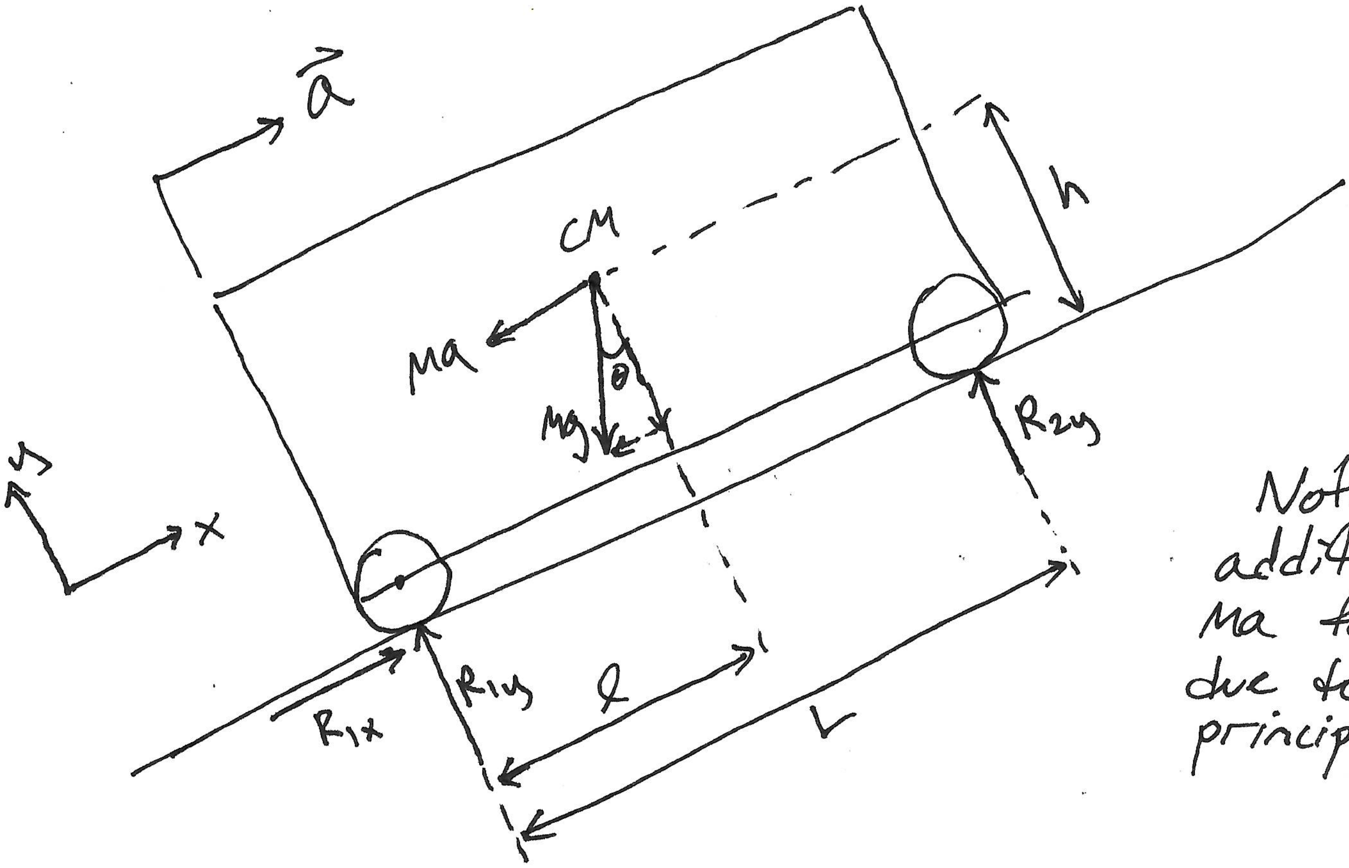
rearrange for spreadsheet:

$$\boxed{\frac{r\mu Mg\left[\cos\theta - \frac{(\cos\theta\, l - \sin\theta\, h)}{L}\right]}{2\tau_{max}} > 1}$$ If true, no slip

c) No tipping during initial acceleration:

- If there is no slip of the wheels, motors can convert τ_{max} to propulsion force

During acceleration:



Note: addition of Ma force due to D'Alembert principle.

Find initial acceleration up the hill:

$$\sum F_x = 0 = R_{1x} - Ma - Mg\sin\theta$$

assuming no slip: $R_{1x} = F_{max} = \dfrac{2\tau_{max}}{r}$

combine: $0 = \dfrac{2\tau_{max}}{r} - Ma - Mg\sin\theta$

$$\Rightarrow a = \frac{\frac{2\tau_{max}}{r} - Mg\sin\theta}{M}$$

Note: when calculating acceleration, check that a is positive → meaning the motors have enough torque to go up the hill.

Find dimensions of vehicle to prevent tipping:

$\Rightarrow$ Vehicle will not tip if moments keeping it on the ground are greater than moments trying to make it tip.

$$\overset{+}{\circlearrowleft} \Sigma M_{①} = 0 = \underbrace{R_{2y}}_{\to 0 \text{ at tipping point}} L - Mg\cos\theta\, l + (Ma + Mg\sin\theta)h$$

Stable if: $Mg\cos\theta\, l > (Ma + Mg\sin\theta)h$

substitute a:

$$g\cos\theta\, l > \left[\frac{2\frac{\tau_{max}}{r} - Mg\sin\theta}{M} + g\sin\theta\right]h$$

rearrange for spreadsheet:

$$\boxed{1 > \frac{h}{lg\cos\theta}\left[\frac{\frac{2\tau_{max}}{r} - Mg\sin\theta}{M} + g\sin\theta\right] \quad \text{If true, no tip}}$$

d) Find values of l and h that make both inequalities true under your robot conditions.

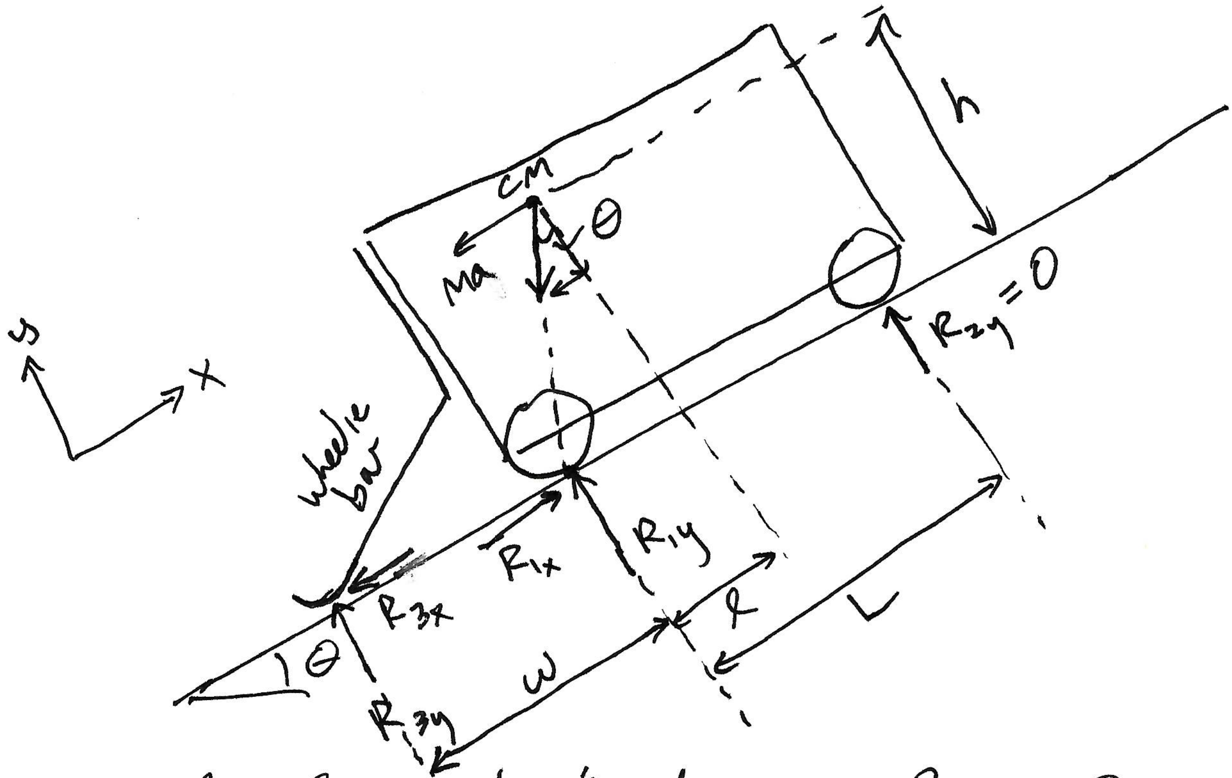
ex) $L = 25\text{cm}$
$M = 5\text{kg}$
$r = 5\text{cm}$
$\theta = 35°$
$\mu = 1$

BO-P5 motors:

$\tau_{max} = 7.2\ \text{kgf}\cdot\text{cm}$ @ 6V
$= 0.72\ \text{N}\cdot\text{m}$

☆ can multiply results of true/false inequalities together to find where both are true (1 if true, 0 if false)

Wheelie Bar

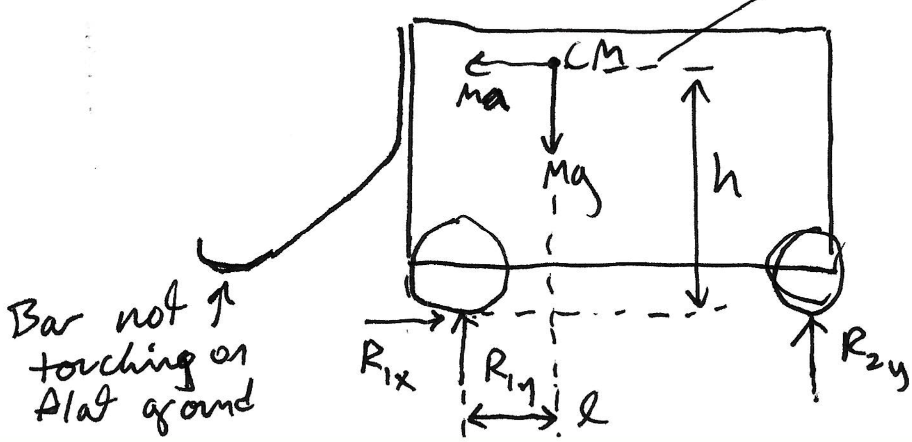


Low friction wheelie bar $\Rightarrow R_{3x} \approx 0$

If w is a long length $\Rightarrow R_{3y}$ will be small (like top fuel dragster front wheels)

$\Longrightarrow$ Most normal force stays on drive wheels so maintain high traction!

Stable on flat ground:



CM is in front of rear wheel contact on flat ground $\Rightarrow$ creates restoring moment for stability if $mg\ell > mah$